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SOME IMPLICATIONS FOR CYCLIC PLASTIC AND VISCOPLASTIC EQUATIONS BASED ON NONPROPORTIONAL LOADING EXPERIMENTS

D.L. McDowell, J. Moosbrugger, M. Doumi
Georgia Institute of Technology
Atlanta, Georgia 30332

E.H. Jordan
University of Connecticut
Storrs, Connecticut 06268

A number of different formulations exist for state variable or "unified" creep-plasticity theory [1-10]. There is, however, a common isothermal framework for many of these models which include backstress, e.g.

$$\dot{\underset{\sim}{\epsilon}}^n = f(||\underset{\sim}{s} - \underset{\sim}{\alpha}||,\kappa)\ (\underset{\sim}{s} - \underset{\sim}{\alpha}) \tag{1}$$

$$\dot{\underset{\sim}{\alpha}} = h_\alpha\ \dot{\underset{\sim}{\xi}} - r_\alpha\ \underset{\sim}{\Omega} \tag{2}$$

$$\dot{\kappa} = h_\kappa ||\dot{\underset{\sim}{\epsilon}}^n|| - r_\kappa \tag{3}$$

where h_α and h_κ are scalar hardening functions, r_α and r_κ are scalar recovery functions, $\underset{\sim}{\alpha}$ is the backstress, κ is the drag stress, $\underset{\sim}{s}$ is deviatoric stress, $\dot{\underset{\sim}{\epsilon}}^n$ is the inelastic strain rate, and $||\dot{\underset{\sim}{\epsilon}}^n|| = [\dot{\underset{\sim}{\epsilon}}^n:\dot{\underset{\sim}{\epsilon}}^n]^{1/2}$.

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1 School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332.

2 Mechanical Engineering, University of Connecticut, Storrs, CT 06268.

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It is usual to first select hardening and recovery functions which encompass relevant uniaxial phenomenological behavior, and then to fit the associated material constants to this data using appropriate multivariate error minimization procedures. There is a somewhat prevalent assertion among existing theories that the directional index for the hardening term in equation (2) is the inelastic strain rate, i.e.

$$\dot{\underset{\sim}{\xi}} = \dot{\underset{\sim}{\epsilon}}^{n} \tag{4}$$

Several theories [2-3, 11-12] include a dynamic recovery term with $\underset{\sim}{\alpha}$ as the directional index, i.e.

$$\dot{\underset{\sim}{\xi}} = \dot{\underset{\sim}{\epsilon}}^{n} - h_{\alpha}^{-1} h_{D} \underset{\sim}{\alpha} || \dot{\underset{\sim}{\epsilon}}^{n} || \tag{5}$$

where h_D is a scalar dynamic recovery function. Uniaxial testing alone is insufficient to validate the directional index of the dynamic recovery term since $\underset{\sim}{\alpha}$ is collinear with $\dot{\underset{\sim}{\epsilon}}^{n}$. This collinearity is also likely responsible for the absence of the dynamic recovery term in many theories.

An important attribute of multiaxial nonproportional loading is the noncollinearity of $\dot{\underset{\sim}{\epsilon}}^{n}$ and $\underset{\sim}{\alpha}$. As will be shown in this paper, the need for the dynamic recovery term can be established from cyclic nonproportional biaxial tests. Furthermore, it is possible to comment on the relative magnitude of the direct hardening and dynamic recovery coefficients and to assess the accuracy of the direct hardening and dynamic recovery directional indices based on selected tests. Axial-torsional experiments conducted with type 304 stainless steel at room temperature and Hastelloy-X at 649°C will be discussed.

Brief Discussion of Results

The need for the dynamic recovery term is evidenced in analysis of the type 304 stainless steel response. As shown in Figure 1, the backstress rate direction much more accurately approaches tangency to the backstress path (assuming constant, rate-independent Mises yield surface radius) with the addition of the dynamic recovery term.

Allowing for rate-dependent response, data from sinusoidal 90° out-of-phase tests on Hastelloy-X were analyzed. Two possibilities were considered. Firstly, the coefficients of the direct and dynamic recovery terms, h_α and h_D in equation (5), were considered scalars. Secondly, they were considered as tensor operators of diagonal form. Several admissible backstress paths were determined by fixed point iteration of an equation reflecting the constraint that backstress must lie along the backward projection of the inelastic strain rate direction from the current stress point. Each assumed initial value of backstress produced a unique, possible backstress path. For each path, the direct hardening and dynamic recovery coefficients were determined by least squares fit to the loading cycle. It was determined that the coefficient of direct hardening is accurately described as a scalar, inferring adequacy of the inelastic strain rate as the directional index. For the dynamic recovery term, however, the data suggest that a tensor-valued coefficient $\underset{\sim}{h_D}$ is appropriate, inferring the inadequacy of backstress $\underset{\sim}{\alpha}$ as a directional index. Refer to Figure 2 for a comparison of the correlation achieved by using scalar and tensor-valued coefficients.

Though limited in number and scope, these results indicate the potential utility of nonproportional biaxial testing in generalization of state variable cyclic viscoplasticity theories.

Conclusions

From cyclic, strain-controlled, nonproportional tests on type 304 stainless steel and Hastelloy-X, the following statements may be made:

1. A dynamic recovery term is essential to properly model the backstress evolution.
2. From analysis of Hastelloy-X data obtained at 649°C, the inelastic strain rate appears to be a satisfactory directional index for direct hardening, but the backstress appears to be an inappropriate directional index of dynamic recovery.
3. Sinusoidal, 90° out-of-phase axial torsional tests can be very useful in aiding determination of backstress evolution functions, including both directional indices and scalar hardening functions, by virtue of the associated approximately constant magnitudes of overstress, inelastic strain rate, and effective stress. Such tests have previously been associated with the study of nonproportional hardening effects but have more far ranging applications.

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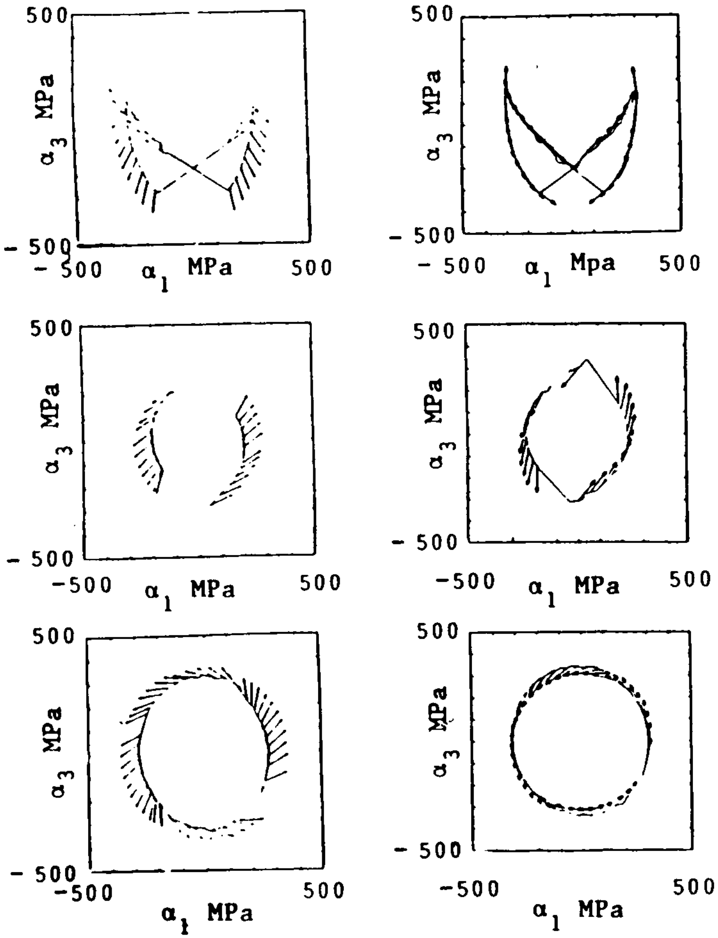

Fig. 1 **Backstress rate directions predicted using (left column) direct hardening only, and (right column) with inclusion of dynamic recovery term for AISI Type 304 Stainless Steel at room temperature. Included are stable cycles from three nonproportional histories. Assumed Mises yield surface radii are (left column) 200 MPa and (right column) 160 MPa.**

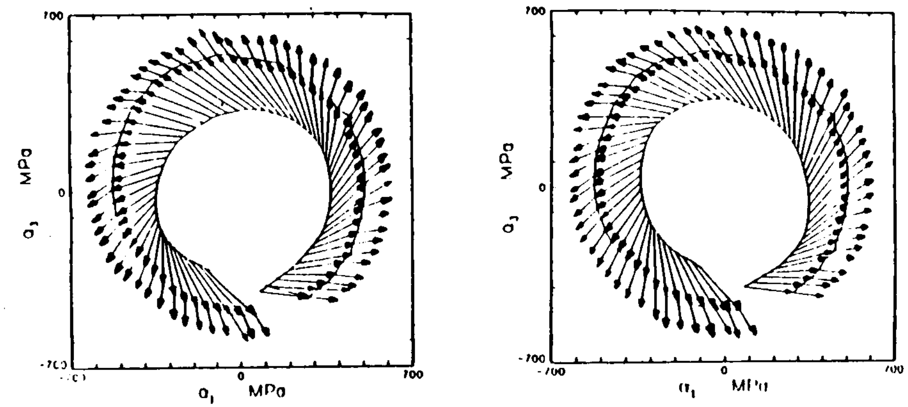

Fig. 2 **Direction of inelastic strain rate (short vectors) and associated overstress vectors $\underline{\sigma}$-$\underline{\alpha}$ for a 90 deg. out-of-phase history on Hastelloy-X at 593°C. Backstress $\underline{\alpha}$ is obtained by Euler integration of (left) scalar direct hardening and dynamic recovery coefficients and (right) tensor-valued coefficients. In each case, the assumed initial value of backstress was 60% of the initial stress. Note the improved collinearity and backstress path shape for the case of tensorial coefficients.**

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